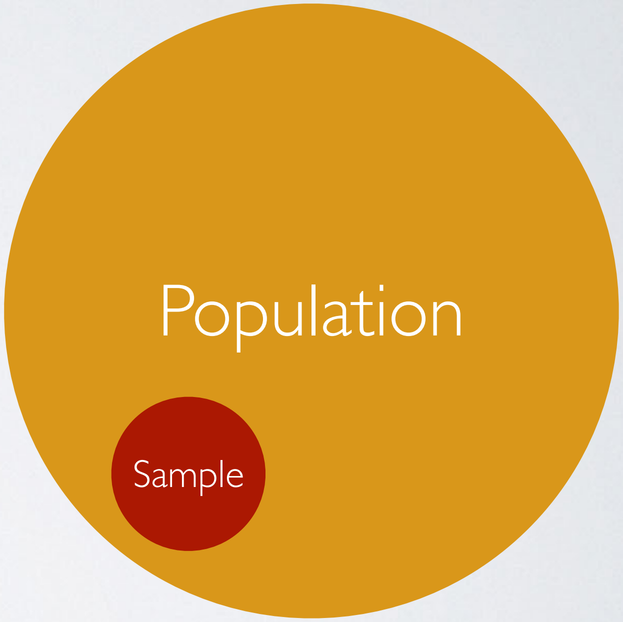
PROBABILITY DISTRIBUTIONS

CHEM 251 SDSU

POPULATIONS & SAMPLES

- In most analyses we are limited to measuring a portion of all the material.
- What we analyze is termed the **sample**.
- The sample is a portion of the **population**.
- As such there is always a question of how well the sample represents the population.
- We can get some insight into the representation by looking at the **distribution** of the measurements we have made.



PROBABILITY DISTRIBUTION TYPES IN CHEMISTRY

- The two most important types of probability distributions in chemistry are:

- **Binomial distributions,** which consist of fixed possible values, such as the values on a die. In chemistry this is manifest principally in isotopic distributions, such as ^{12}C, ^{13}C, and ^{14}C.

- **Normal distributions**, which consist of a continuum of all possible values, such as the percent composition of a mixture of water and ethanol, it can range from 0% to 100% ethanol.

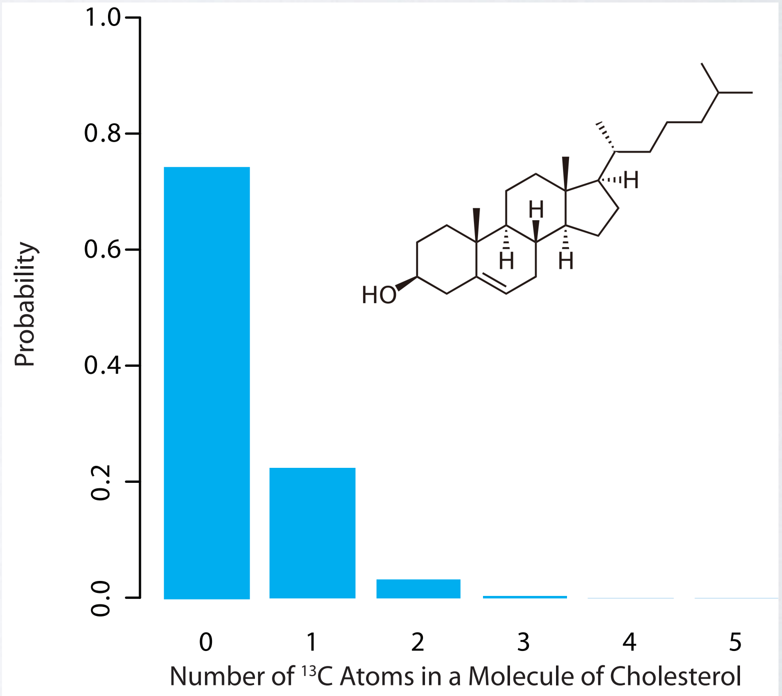
BINARY DISTRIBUTIONS

- Binary distributions have fixed outcome values.

$$P(X,N) = \frac{N!}{X!(N-X)!} \times p^X \times (1-p)^{N-X}$$

- The probability of any value occurring can be calculated by *P(X,N)*.

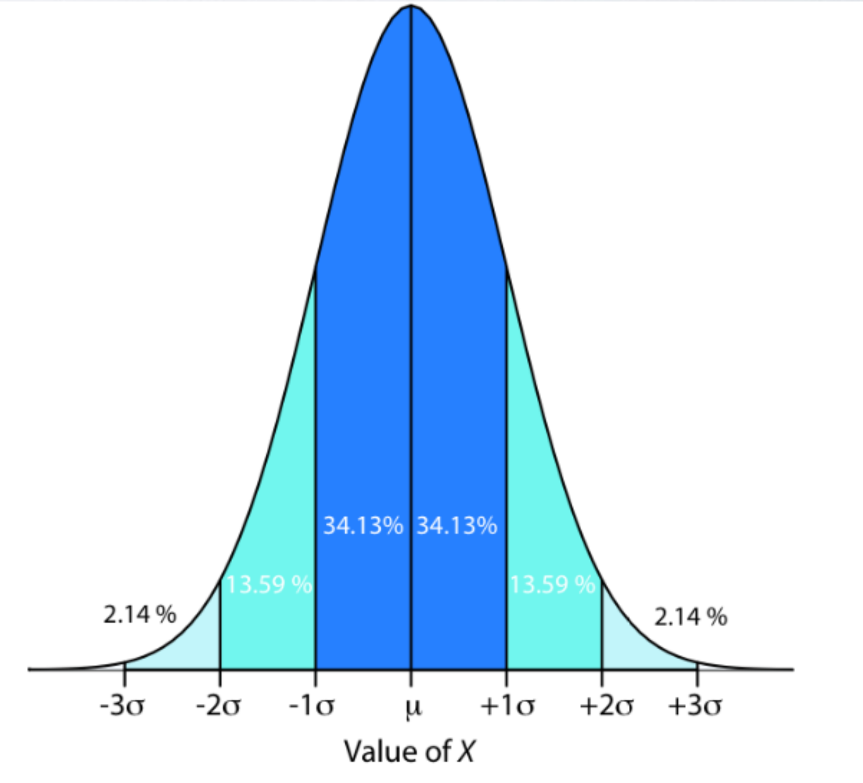
- Where:
    - *X* is number of occurrences
    - *N* is the number of trials
    - *p* is the event's probability (e.g. ^{12}C=98.89%, ^{13}C=1.11%)



NORMAL DISTRIBUTION

- Normal distributions can have any possible value, between the upper and lower limits.
- The probability of any value (*X*) occurring is based off the equation to the right.
- Where:
  - σ is the standard deviation
  - μ is the population mean

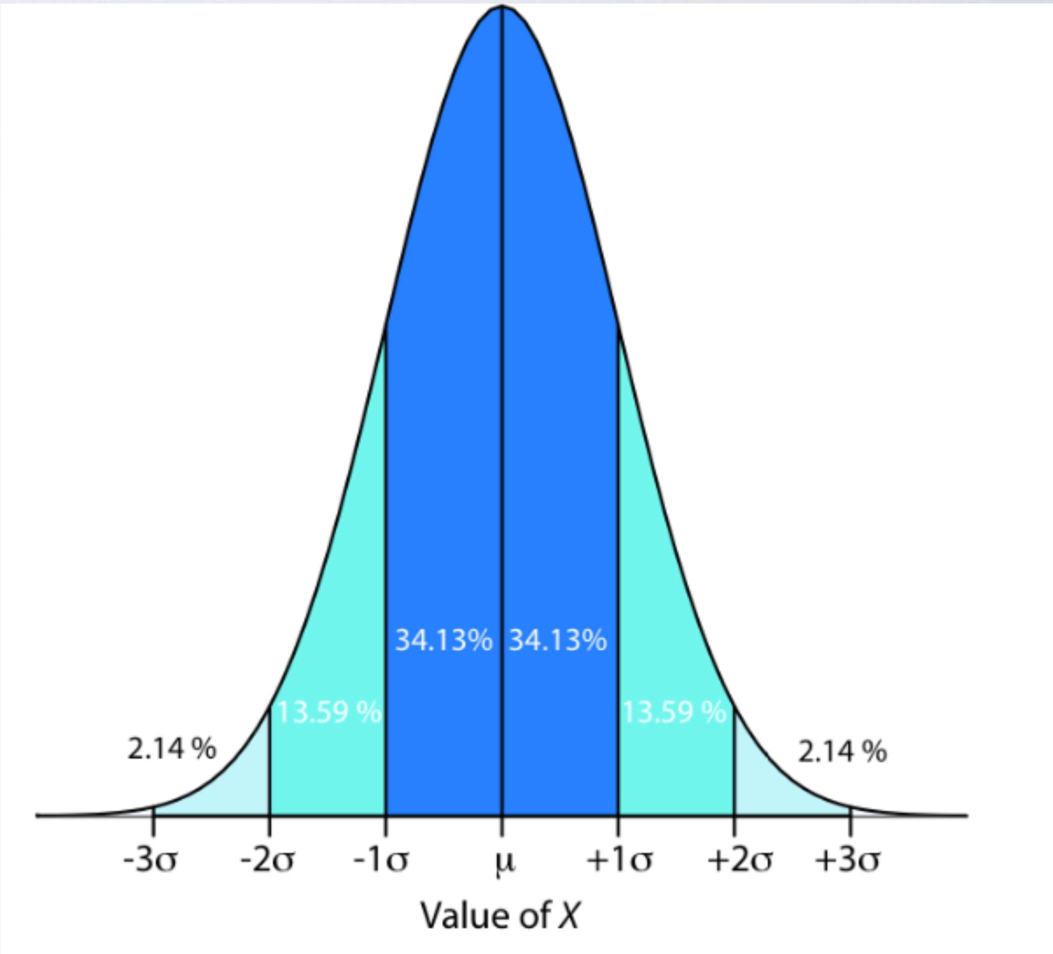
$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(X-\mu)^2}{2\sigma^2}}$$



NORMAL DISTRIBUTIONS

- The probability of any member of the population being within a given range of the mean can be expressed in terms of the standard deviation (**σ**).

- This can be used to determine the probability of any member falling within a given range.



INFLUENCE OF STANDARD DEVIATIONS

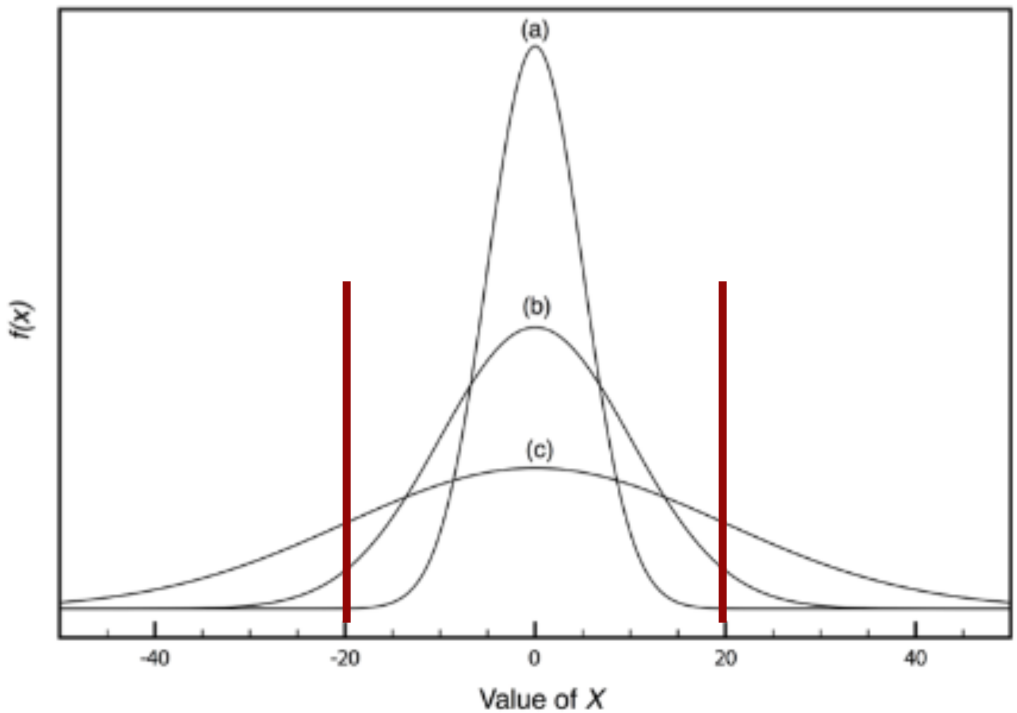


Figure 4.7 Normal distribution curves for:

(a) $\mu = 0; \sigma^2 = 25$

(b) $\mu = 0; \sigma^2 = 100$

(c) $\mu = 0; \sigma^2 = 400$

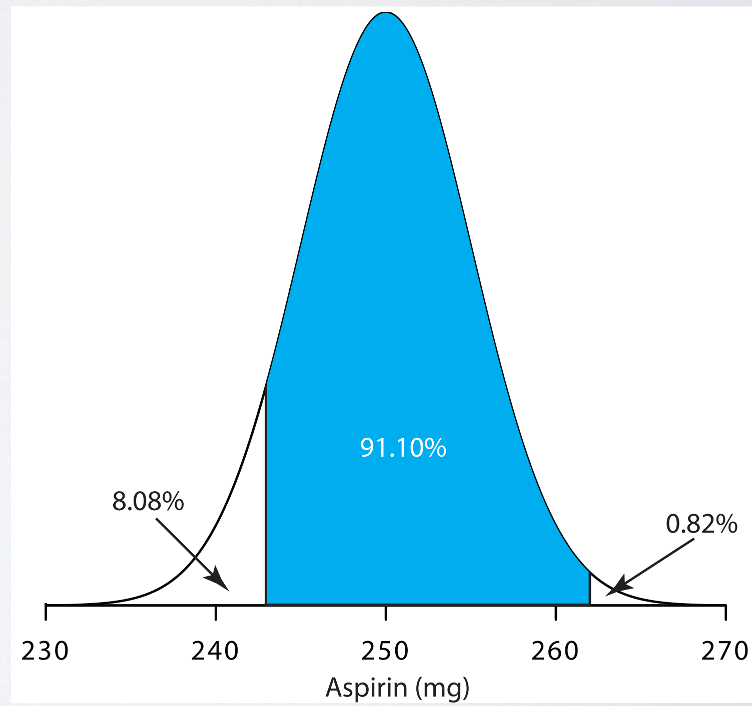
For distribution (a) **99.99%** of all members of the population within ± 20 of the mean.
For distribution (c) only **68.28%** is within ± 20 units of the mean.

PROBABILITY OF A GIVEN RANGE

$$z = \frac{X - \mu}{\sigma}$$

- The nature of a normal distribution allows us to calculate the probability of making replicate measurements that within a given range.

- Once we define the upper and lower limits of our region we can use the values of z to determine the probability of the measurement.



SAMPLE PROBLEM

What percentage of Aspirin tablets would have between 254 and 258 mg of Aspirin if the population mean (μ) is 250 mg of Aspirin and the standard deviation (σ) is 5 mg?

Table of z values.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4365	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4502	0.4013	0.3974	0.3396	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0466	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183